

THE CONCEPT OF ELEMENTARY INVARIANT OPERAND IN MOMENT  
METHOD -APPLICATION TO THE ELECTROMAGNETIC OPTIMIZATION OF  
SHIELDED PLANAR DISCONTINUITIES.

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### ABSTRACT

This paper presents a new concise way to run the numerical information required to solve an electromagnetic problem using the method of moment. As a consequence, it enables the rigorous analysis of large circuits and the introduction of electromagnetic optimization.

### INTRODUCTION

Electromagnetic simulators are now widely used to rigorously analyze microwave circuits. Most of those which are dedicated to planar technologies rely on the technique of integral equations solved by the method of moment. Unfortunately, electromagnetic simulators still remain auxiliary tools whose only task is to punctually supply circuit simulators in the rigorous description of complex local phenomena's (shielding effect, parasitic coupling, radiation, ...). Due to the computer resources it requires, a global analysis is not really possible yet and, consequently, only small parts of complex circuits can be analyzed electromagnetically. Moreover, the electromagnetic synthesis of circuits is not available yet even though it would be a precious means of optimizing their dimensions and their very shape in critical configurations. An interesting attempt has been reported recently to optimize discontinuities using the TLM method [1] but it involves a massively parallel computer.

This paper proposes a complete procedure to electromagnetically optimize simple discontinuities using the method of moment. This approach relies on the intensive exploitation of all real and pseudo invariances in the studied structure (the concept of pseudo invariance will be explained latter). More practically, it involves the determination of what we call elementary invariant operands (EIOs). EIOs are defined as the smallest independent subdivisions of the information required at the last step of the analysis of the structure. They altogether contain all the information while no redundancy exists between them. The use of EIOs provides the most concise way of running the information. As a consequence, it permits to reduce space requirements and computation time. Moreover, it will be shown that once EIOs have been computed for a given structure, they can be recombined instantaneously to analyze any other structure included in the first one. Thanks to this very important property, this approach appears particularly well-suited to optimization problems.

This paper first presents the calculation of EIOs in the case of a shielded planar structures. This example was chosen because it involves a non obvious definition of EIOs as no real invariance exists, due to the presence of the metallic box. As an application, the global study of a complete microstrip Wilkinson power divider is then presented. A first trial of optimization is also proposed for a suspended microstrip stub.

### THEORY

Consider the structure shown on fig. 1. It consists of a parallelipedic metallic box partially filled with a dielectric substrate. The upper face of the dielectric layer supports a metallization of any shape. This generic configuration can be used to modelize shielded microstrip structures ( $h_1=0$ ) or suspended microstrip structures ( $h_1>0$ ). Additional layers could be introduced with no loss of generality.

Once the metallization has been divided into rectangular cells, moment method can be applied to convert an adequate integral equation into a linear system [2]:

$$\begin{bmatrix} Z^{xx} & Z^{xy} \\ Z^{yx} & Z^{yy} \end{bmatrix} \begin{bmatrix} I^x \\ I^y \end{bmatrix} = \begin{bmatrix} V^x \\ V^y \end{bmatrix} \quad (1)$$

where:

$Z^{uv}$  ( $u=x,y$  and  $v=x,y$ ) is a submatrix of the generalized impedance matrix,

$I^v$  is the vector of the unknown currents directed according to  $v$ ,

$V^u$  is the vector of the excitation voltage applied according to  $u$ .

In this equation, each element  $Z_{pq-kl}^{xx}$  of the first submatrix corresponds to the contribution of the source current defined by direction  $x$  and coordinates  $(k,l)$  to the observer voltage defined by direction  $x$  and coordinates  $(p,q)$ :

$$Z_{pq-kl}^{xx} = \sum_{m=0}^M \sum_{n=0}^N f_{mn}^{xx}(a,b,h,\epsilon_r,f) \cos(k_x x_k) \cos(k_x x_p) \sin(k_y y_l) \sin(k_y y_q) \quad (2)$$

where:

$k_x = m\pi/a$  and  $k_y = n\pi/b$ ,

$x_k = k \, d_x$ ,  $x_p = p \, d_x$ ,  $y_l = l \, d_y$  and  $y_q = q \, d_y$ ,

$d_x$  and  $d_y$  are the dimensions of a cell,

$f_{mn}^{xx}$  is a function of the geometry and of the frequency,

$M, N$  are integer coefficients used to truncate infinite summations.

Let:

$$EIO(i,j) = \frac{1}{4} \sum_{m=0}^M \sum_{n=0}^N f_{mn}^{xx}(a,b,h,\epsilon_r,f) \cos(ik_x d_x) \sin(jk_y d_y) \quad (3)$$

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(2) can be rewritten:

$$Z_{pq-kl}^{xx} = \text{EIO}(k-p, l-q) - \text{EIO}(k-p, l+q+1) \\ + \text{EIO}(k+p, l-q) - \text{EIO}(k+p, l+q+1) \quad (4)$$

Thanks to this transformation, each element of the submatrix is now expressed as a linear combination of four elementary operands. Each EIO is involved in the determination of several submatrix element which means the total number of double summation to be computed considerably decreases. Typically, if the metallization is a  $N_x \times N_y$  cells rectangle, the first submatrix contains  $[(N_x - 1) N_y]^2$  elements while it only involves  $2 N_y (2 N_x - 1)$  EIOs.

This kind of decomposition is well known in open structures where each submatrix element only depends on the distance between the source and the observer (dependence in  $k-p$  and in  $l-q$ ), thanks to the radial invariance of the correspondent Green's function [3]. It is extended here to closed structures where this invariance doesn't exist. In that case, equation (4) shows that each submatrix element not only depends on the distance between the source and the observer (dependence in  $k-p$  and in  $l-q$ ) but also on the distance between the box and the center of the segment connecting the source to the observer (dependence in  $k+p$  and  $l+q$ ). That is what we call pseudo invariance. The same kind of decomposition is of course also done with the elements of the three other submatrix.

Equation (3) also shows that  $f_{mn}^{xx}$  is perfectly defined when the metallic box ( $a, b$ ), the substrate ( $h, \epsilon_r$ ), its position ( $h_1$ ) and the frequency ( $f$ ) have been chosen for a given application. This means that once the EIOs have been calculated for a large rectangle metallization, the generalized matrix impedance representing any metallization included in this rectangle can be constructed instantaneously using (4).

## APPLICATIONS

Thanks to this technique, moment method is easily applied to a complete Wilkinson power divider (fig. 2), centered in a metallic box. Validation of this method of moment has been done with simpler structures in previous papers [4-5] and will not be discussed here. In this example, the relative disagreement for higher frequencies (especially for  $S_{23}$ ) only demonstrates the limitations of a non electromagnetic analysis.

Fig. 4 presents a first illustration of the optimization capabilities of this approach. The desired goal is to design an open-ended stub whose resonant frequency is 30 GHz. This stub is located on a shielded microstrip line. The maximum surface allowed to the stub is specified on the meshing grid. Practically, it translates the designer's constraints. Numerically, it corresponds to the larger metallization to be analyzed (line excluded) and so defines the number of EIOs to be evaluated. Once the EIOs have been evaluated, any configuration included in this surface can be analyzed rapidly (it only requires a matrix inversion). An optimization procedure controls the choice of the configurations to be analyzed. Fig. 5 sums up the evolution of the optimization

procedure. It only requires 9 iterations and 1.9s CPU time on a HP735 workstation to achieve a -28dB resonant stub.

## CONCLUSION

This paper proposes a new concept to run the numerical information using the method of method. It relies on the research of the smallest independent subdivisions of information, the so called EIOs. An application is given for shielded structures where the problem is crucial as no invariance exists. As an illustration, a procedure of electromagnetic optimization is presented. This approach is expected to provide a powerful means of performing the synthesis of microwaves and millimeter waves planar circuits.

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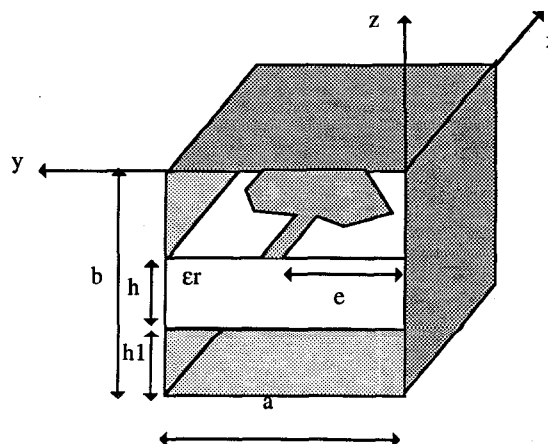


Fig.1 : studied structure

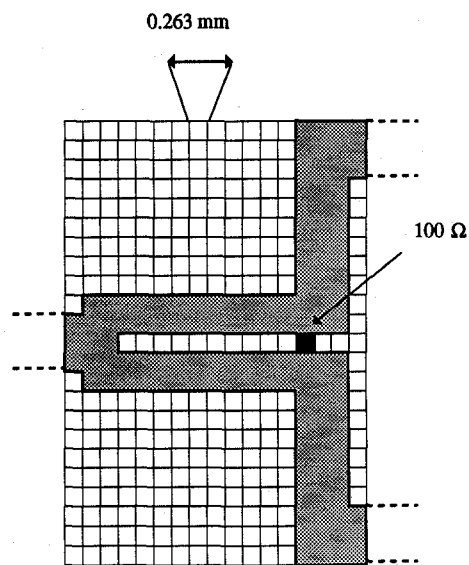
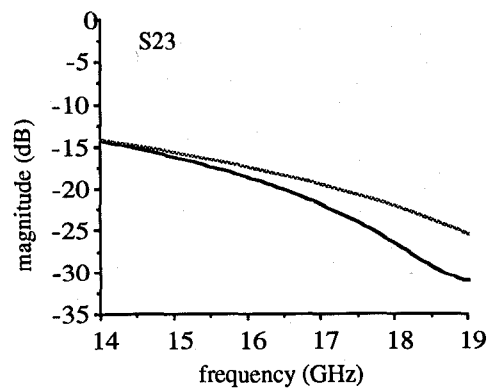
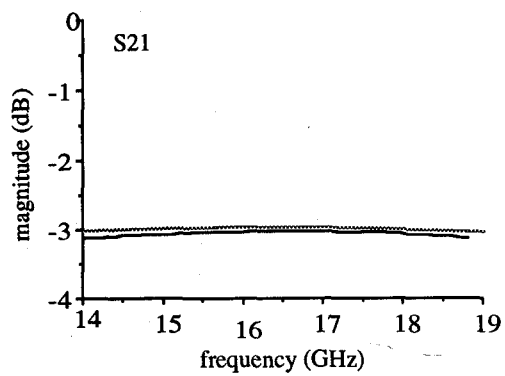
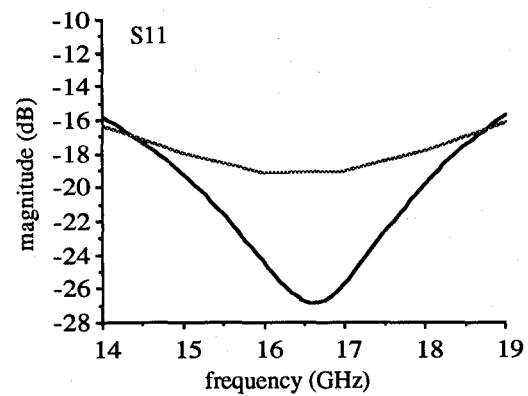


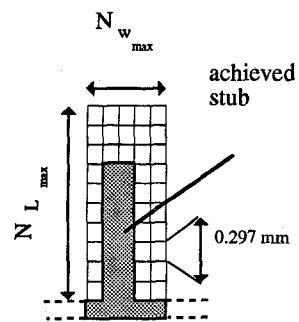
Fig. 2 : studied Wilkinson

$a = 7.112 \text{ mm}$   
 $b = 3.556 \text{ mm}$   
 $\epsilon_r = 2.2$   
 $h_1 = 0 \text{ mm}$   
 $h = 0.254 \text{ mm}$   
 $e = 3.156 \text{ mm}$

Fig. 3 : scattering parameters

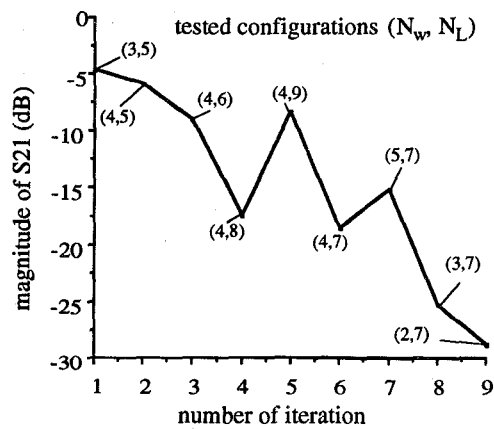
— MoM  
 - - - circuit simulator





$a = 4.45 \text{ mm}$   
 $b = 1.78 \text{ mm}$   
 $\epsilon_r = 2.2$   
 $h_1 = 0.636 \text{ mm}$   
 $h = 0.254 \text{ mm}$   
 $e = 0.526 \text{ mm}$   
 $f = 30 \text{ GHz}$

**Fig. 4** : structure to be optimized with maximum dimensions and achieved stub



**Fig. 5** : evolution of the optimization procedure